

1. Last time we showed for a bare proton in a magnetic field B_z has energy $E_{\pm\frac{1}{2}} = \mp\frac{1}{2}\hbar\gamma B_z$ and partition function $Q = 2 \cosh\left(\frac{\beta\hbar\gamma B_z}{2}\right)$, where $\cosh x = \frac{e^x + e^{-x}}{2}$. Calculate the average energy for this system by method of partial derivatives.

2. Given that the heat capacity at constant volume is defined as $C_V = \left(\frac{\partial U}{\partial T}\right)_V$ and that the internal energy is given by $\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P$, derive the following relationship:

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T \left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

For this class, mixed partial derivatives always commute (Clairaut's theorem):

$$\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x}\right)$$

3. In a few lectures we will learn that the partition function for a diatomic gas is $Q(N, V, \beta) = \frac{[q(V, \beta)]^N}{N!}$, where $q(V, \beta) = \left(\frac{2\pi m}{h^2\beta}\right)^{3/2} V \cdot \frac{8\pi^2 I}{h^2\beta} \cdot \frac{e^{-\beta h\nu/2}}{1 - e^{-\beta h\nu}}$. Use this information to derive this expression for the average energy per mole of a diatomic ideal gas:

$$\bar{U} = \frac{5}{2}RT + \frac{N_A h\nu}{2} + \frac{N_A h\nu e^{-\beta h\nu}}{1 - e^{-\beta h\nu}}$$

Homework Problem 4

1. Consider a system of N identical, noninteracting particles with partition function $Q(N, V, T) = \frac{[q(V, T)]^N}{N!}$ where the single-particle partition function is given by $q(V, T) = f(T) \cdot V$. Using the thermodynamic relation $P = k_B T \left(\frac{\partial \ln Q}{\partial V} \right)_{T, N}$ show that the resulting equation of state is the ideal gas law.