

1. Later on, we will learn that the single molecule rotational partition function is given by:

$$q_{\text{rot}}(T) = \frac{\pi^{1/2}}{\sigma} \left(\frac{8\pi^2 I_A k_B T}{h^2} \right)^{1/2} \left(\frac{8\pi^2 I_B k_B T}{h^2} \right)^{1/2} \left(\frac{8\pi^2 I_C k_B T}{h^2} \right)^{1/2}$$

Show that the rotational contribution to molar heat capacity is given by $\bar{C}_{v,\text{rot}} = \frac{3k_B}{2}$. Treat $Q = q_{\text{rot}}$

2. A system of N harmonic oscillators is called an Einstein Solid. The partition function is:

$$Q = \left[\frac{1}{1 - e^{-\beta\hbar\omega}} \right]^N$$

(a) Show that the internal energy is:

$$U = \frac{N\hbar\omega}{e^{\beta\hbar\omega} - 1}$$

(b) Show that the heat capacity is:

$$C_V = Nk_B(\beta\hbar\omega)^2 \frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega} - 1)^2}$$

Homework Problem 5

1. Consider a system of N independent, distinguishable particles that have two possible quantum states $\varepsilon_0 = 0$ and $\varepsilon_1 = \varepsilon$. The partition function for this system is given by:

$$Q = q^N = (1 + e^{-\beta\varepsilon})^N$$

Show the molar heat capacity is given by: $\bar{C}_v = R(\beta\varepsilon)^2 \frac{e^{-\beta\varepsilon}}{(1+e^{-\beta\varepsilon})^2}$