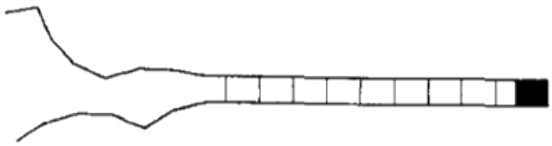
1. Today, we will use statistical mechanics to develop a zipper model commonly used in modeling DNA.



Consider a zipper model with the following properties:

- The zipper is at temperature T and consists of N links.
- Each link can be either closed with energy 0 or open with energy ε.
- When a link is closed, there is only one possible configuration. However, when a link is open, it can rotate freely, resulting in a degeneracy g.
- The zipper can only unzip from one end, meaning that link i can only be open if all preceding links $(0, 1, 2, \ldots, i-1)$ are also open.
- The final link is always closed to prevent the DNA from fully disconnecting.

Show that the average number of open links is given by:

$$\langle i \rangle = \frac{Nx^N}{x^N - 1} - \frac{x}{x-1}$$

where $x \equiv ge^{-\beta\varepsilon}$

Useful formula: The sum of a geometric series is given is:

$$\sum_{a=0}^{N} r^a = \frac{1 - r^{N+1}}{1 - r}, \quad \text{for } r \neq 1$$

Homework Problem 6

1. The partition function of a mixture of two ideal gases is given as:

$$Q(N_1, N_2, V, T) = \frac{[q_1(V,T)]^{N_1}}{N_1!}\frac{[q_2(V,T)]^{N_2}}{N_2!}$$

where $q_j(V,T) = \left(\frac{2\pi m_j k_{\mathrm{B}} T}{h^2}\right)^{3/2} V$.
Show that energy is an additive quantity, namely:

$$\langle E \rangle = \frac{3}{2}(N_1 + N_2)k_B T$$