

1. Starting from the total differential of $S(T, P)$, show that:

$$\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial S}{\partial T}\right)_P - \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_V$$

Total differential:

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

Dividing through by dT at constant V :

$$\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial S}{\partial T}\right)_P + \left(\frac{\partial S}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V$$

Applying the Maxwell relation $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$:

$$\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial S}{\partial T}\right)_P - \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_V$$

2. Starting from:

$$C_P - C_V = T \left(\frac{\partial S}{\partial T}\right)_P - T \left(\frac{\partial S}{\partial T}\right)_V$$

Show that:

$$C_P - C_V = T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_V$$

You can use your result from 1 to help solve this problem.

$$C_P - C_V = T \left(\frac{\partial S}{\partial T}\right)_P - T \left(\frac{\partial S}{\partial T}\right)_V$$

Using 1 to help solve this problem

$$\begin{aligned} &= T \left(\frac{\partial S}{\partial T}\right)_P - T \left(\frac{\partial S}{\partial T}\right)_P - T \left(\frac{\partial S}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V \\ &= -T \left(\frac{\partial S}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V \end{aligned}$$

Applying the Maxwell relation $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$:

$$C_P - C_V = T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_V$$

3. Starting from:

$$\left(\frac{\partial T}{\partial V}\right)_U \left(\frac{\partial V}{\partial U}\right)_T \left(\frac{\partial U}{\partial T}\right)_V = -1$$

Show that:

$$\left(\frac{\partial T}{\partial V}\right)_U = \frac{1}{C_V} \left[P - T \left(\frac{\partial P}{\partial T}\right)_V \right]$$

Instead of using your result from activity 20, determine a relation from $dU = T dS - P dV$.

Rearrange to match RHS

$$\begin{aligned} \left(\frac{\partial T}{\partial V}\right)_U &= - \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial T}{\partial U}\right)_V \\ &= - \left(\frac{\partial U}{\partial V}\right)_T \cdot \frac{1}{C_V} \end{aligned}$$

Take $\left(\frac{\partial}{\partial V}\right)_T$ of both sides of $dU = T dS - P dV$:

$$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial S}{\partial V}\right)_T - P$$

Apply the Maxwell relation from dA : $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P$$

Substitute back:

$$\left(\frac{\partial T}{\partial V}\right)_U = \frac{1}{C_V} \left[P - T \left(\frac{\partial P}{\partial T}\right)_V \right]$$

Homework Problem 21

1. Starting from:

$$dH = T dS + V dP$$

Show that:

$$\left(\frac{\partial H}{\partial V}\right)_T = \frac{V - T \left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial V}{\partial P}\right)_T}$$